Score:

Name: Solutions

Section (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM365 – Test 3 – Fall 2011

x	$f(x)$
0	-2.5
1	2
3	14

1. (15 pts) Using the data table on the right:
   a. Find the Lagrange interpolating polynomial.
   b. Use your result for estimate $f(2)$.
   c. If $|f'''(x)|$ is bounded by .25 on the interval [0,3] obtain a bound on the error for the approximation $f(2)$.

$L_{2,0} = \frac{(x-1)(x-3)}{(0-1)(0-3)} = \frac{1}{3}(x^2-4x+3)$

$L_{2,1} = \frac{(x)(x-3)}{(1-0)(1-3)} = -\frac{1}{2}(x^2-3x)$

$L_{3,1} = \frac{(x)(x-1)}{(3-0)(3-1)} = \frac{1}{6}(x^2-x)$

$-2.5L_{2,0} + 2L_{2,1} + 14L_{3,1} =$

a) $\boxed{\frac{1}{2}x^2 + 4x - 2.5 = P_2(x)}$

$\Rightarrow f(2) = \frac{1}{2}(4) + 4(2) - 2.5 = 2+8-2.5$

b) $\Rightarrow \boxed{f(2) = 7.5}$

c) $|E| \leq \left| \frac{f'''(\xi)}{3!}(2-0)(2-1)(2-3) \right| \leq \boxed{.08333}$ (.25)

No marks on this table		
TH (10 pts)		
1 (15 pts)		
2 (15 pts)		
3 (15 pts)		
4 (15 pts)		
5 (15 pts)		
6 (15 pts)		
cumm.		

2. (15 pts) Using the data table on the right:
   a. Construct a divided difference table.
   b. Find the Newton interpolating polynomial.
   c. Use your result to estimate $f(1)$.

x	$f(x)$
-1	-2
0	-3
2	7

(a)

x	$f(x)$		
-1	-2		
		-1	
0	-3		2
		5	
2	7		

(b)

$$P_2(x) = -2 - 1(x-(-1)) + 2(x-(-1))(x)$$

$$= -2 - 1(x+1) + 2(x^2+x)$$

$$= -2 - \cancel{x} - 1 + 2x^2 + \cancel{2}x \Rightarrow \boxed{P_2(x) = 2x^2 + x - 3}$$

$$f(1) \approx 2 + 1 - 3 \Rightarrow \boxed{f(1) \approx 0}$$

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3. (15 pts) Use matrices to find the interpolating polynomial for the data on the right.

x	$f(x)$
-2	4
-1	3
0	2
1	7

$$\boxed{a_0 + a_1 x + a_2 x^2 + a_3 x^3 = P_n(x)}$$

$$a_0 + a_1(-2) + a_2(-2)^2 + a_3(-2)^3 = 4$$

$$a_0 + a_1(-1) + a_2(-1)^2 + a_3(-1)^3 = 3$$

$$a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 = 2$$

$$a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 = 7$$

$$\underbrace{\begin{bmatrix} 1 & -2 & 4 & -8 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 4 \\ 3 \\ 2 \\ 7 \end{bmatrix}}_{\vec{b}}$$

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$$A\vec{x} = \vec{b} \Rightarrow \vec{x} = A^{-1}b$$

$$\Rightarrow a_0 = 2, \ a_1 = 1, \ a_2 = 3, \ a_3 = 1$$

$$\Rightarrow \boxed{P_3(x) = x^3 + 3x^2 + x + 2}$$

4. (15 pts) Find the line of best fit for the data on the right. Estimate $f(2)$

x	$f(x)$
-2	4
-1	3
0	2
1	7

$$ax + b \stackrel{?}{=} y$$

$$\Rightarrow \begin{aligned} -2x + b &= 4 \\ -x + b &= 3 \\ +0x + b &= 2 \\ x + b &= 7 \end{aligned}$$

$$\Rightarrow \underset{\vec{A}}{\begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}} \underset{x}{\begin{bmatrix} a \\ b \end{bmatrix}} = \underset{\vec{b}}{\begin{bmatrix} 4 \\ 3 \\ 2 \\ 7 \end{bmatrix}} \Rightarrow \quad A\vec{x} = b \Rightarrow A^T A \vec{x} = A^T b$$

$$\Rightarrow \vec{x} = (A^T A)^{-1} A^T b$$

you can do this by hand

① $A^T A = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$

② $(A^T A)^{-1} = \frac{1}{20} \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 1/5 & 1/10 \\ 1/10 & 3/10 \end{bmatrix}$

③ $A^T b = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} -4 \\ 16 \end{bmatrix}$

④ $(A^T A)^{-1} A^T b = \begin{bmatrix} .2 & .1 \\ .1 & .3 \end{bmatrix} \begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{matrix} .8 \\ 4.4 \end{matrix}$

$$\boxed{y = .8x + 4.4}$$

$$y(2) = .8(2) + 4.4 = 6$$

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5. (15 pts) Find the error for the following approximating formula:

$$f''(x) = \frac{1}{h^2}(f(x) - 2f(x+h) + f(x+2h))$$

$f(x) = f(x)$

$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(\xi)$

$f(x+2h) = f(x) + 2h f'(x) + \frac{4h^2}{2!} f''(x) + \frac{8h^3}{3!} f'''(\xi)$

$$|E| = \left| \frac{-2}{h^2} \frac{h^3}{3!} f'''(\xi) + \frac{1}{h^2} \frac{8h^3}{3!} f'''(\xi) \right|$$

$$= \left| \left(\frac{-2}{6} h + \frac{8}{6} h \right) f'''(\xi) \right|$$

$$= \left| \frac{+6}{6} h f'''(\xi) \right| = \boxed{\left| h f'''(\xi) \right|}$$

6. (15 pts) Fill in the missing values from the given extrapolation table. Indicate the order of approximation in each successive column. Additional, state explicitly the equation that you used to perform each extrapolation.

$f'(x)_{Approx} = \dfrac{f(x+h) - f(x-h)}{2h}$			
h	$o(h^2)$	$O(h^4)$	$O(h^6)$
.25	0.0833484109		
.5	0.0833937304	.0833333045	
1	0.0835763201	.0833328671	.0833333336

$$\frac{2^2 D_{h/2} - D_h}{2^2 - 1}$$

$$\boxed{\frac{4D_{h/2} - D_h}{3}}$$

$$\frac{2^4 D_{h/2} - D_h}{2^4 - 1}$$

$$\boxed{\frac{16D_{h/2} - D_h}{15}}$$

$$\frac{1}{3} x^{-2/3} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12} = .08\overline{3}$$