Score:

Name:
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM365 – Numerical Computing – Quiz 9– Section 6.2
Numerical Differentiation – Take Home

1. Find the 3 point forward difference approximation, i.e. use $f(x), f(x+h), f(x+2h)$, for the first derivative $f'(x)$. Include the error term.

$$a \times \quad f(x) = a f(x)$$
$$b \times \quad f(x+h) = b f(x) + bh f'(x) + b\frac{h^2}{2} f''(x) + b\frac{h^3}{3!} f'''(\xi)$$
$$c \times \quad f(x+2h) = c f(x) + c2h f'(x) + c\frac{4h^2}{2} f''(x) + c\frac{8h^3}{3!} f'''(\xi)$$

Ignore for know

$$a + b + c = 0$$
$$bh + 2ch = 1 \Rightarrow \quad b + 2c = 1/h \Rightarrow -4c + 2c = \quad -2c = 1/h \Rightarrow \boxed{c = -1/2h}$$
$$\frac{bh^2}{2} + 2ch^2 = 0 \quad\quad \frac{b}{2} + 2c = 0 \Rightarrow \boxed{b = -4c} \Rightarrow b = 2/h$$

$$\Rightarrow a + \frac{2}{h} - \frac{1}{2h} = 0 \Rightarrow \boxed{a = -\frac{3}{2h}}$$

$$\Rightarrow f'(x) = -\frac{3}{2h} f(x) + \frac{2}{h} f(x+h) - \frac{1}{2h} f(x+2h)$$

$$\boxed{f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}}$$

$$E = \left(b\frac{h^3}{3!} + c\frac{8h^3}{3!}\right) f'''(\xi) = \left(\frac{2}{h}\frac{h^3}{3!} - \frac{1}{2h}\frac{8h^3}{3!}\right) f'''(\xi)$$
$$= \left(\frac{2}{6} - \frac{4}{6}\right) h^2 f'''(\xi) = \boxed{-\frac{1}{3}h^2 f'''(\xi)}$$

(Question 2 on Back!)

4-1

$\frac{3}{12}$

2. Use the same three points to get an approximations for $f''(x)$. Include error term.

same set of equations w/slight change

$a+b+c=0 \quad \Rightarrow a-\frac{2}{h^2}+\frac{1}{h^2}=0 \Rightarrow a=\frac{1}{h^2}$

$bh+2ch=0 \quad b+2c=0 \Rightarrow b=-2c \Rightarrow \boxed{b=-2/h^2}$

$\frac{bh^2}{2}+2ch^2=1 \Rightarrow \quad \frac{b}{2}+2c=1/h^2 \Rightarrow -c+2c=1/h^2$

$\Rightarrow \boxed{c=1/h^2}$

$$\therefore f''(x) \approx \frac{1}{h^2}f(x) - \frac{2}{h^2}f(x+h) + \frac{1}{h^2}f(x+2h)$$

or $$\boxed{f''(x) = \frac{f(x) - 2f(x+h) + f(x+2h)}{h^2}}$$

$$E = \left(b\frac{h^3}{3!} + c\frac{8h^3}{3!}\right)f'''(\xi)$$

$$= \left(\frac{-2}{h^2}\frac{h^3}{3!} + \frac{1}{h^2}\frac{8h^3}{3!}\right)f''(\xi)$$

$$= \left(\frac{6}{3!}\right)h\,f''(\xi) \Rightarrow \boxed{E = h\,f'''(\xi)}$$