Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM365 – Numerical Computing – Quiz 9B– Section 6.2
Numerical Differentiation – Take Home

1. Find the 3 point forward difference approximation, i.e. use $f(x), f(x+h), f(x+3h)$, for the first derivative $f'(x)$. Include the error term.

$$f(x) = f(x)$$

$$f(x+h) = f(x) + \frac{h^1}{1!} f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(\xi)$$

$$f(x+2h) = f(x) + \frac{3h}{1!} f'(x) + \frac{9h^2}{2!} f''(x) + \frac{27h^3}{3!} f'''(\xi)$$

$$\Rightarrow \begin{bmatrix} 0^0 & 1^0 & 2^0 \\ 0^1 & 1^1 & 2^1 \\ 0^2 & 1^2 & 2^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1/h \\ 0 \end{bmatrix} \Rightarrow$$

$$a = -\frac{4}{3h} = \frac{-8}{6h}$$
$$b = \frac{3}{2h} \qquad \frac{9}{6h}$$
$$c = \frac{-1}{6h} \qquad \frac{-1}{6h}$$

$$\Rightarrow f'(x) = \frac{-8f(x) + 9f(x+h) - f(x+2h)}{6h} + \frac{h^2}{2} f'''(\xi)$$

$$E = \frac{3}{2h} \frac{h^3}{6} f'''(\xi) - \frac{1}{6h} \frac{27h^3}{6!} f'''(\xi)$$

$$= \frac{h^2}{4} f'''(\xi) - \frac{3h^2}{4} f'''(\xi) = -\frac{h^2}{2} f'''(\xi)$$

(Question 2 on Back!)

2. Use the same three points to get an approximations for $f''(x)$. Include error term.

$$\begin{bmatrix} 0^0 & 1^0 & 3^0 \\ 0^1 & 1^1 & 3^1 \\ 0^2 & 1^2 & 3^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2/h^2 \end{bmatrix} \implies \begin{array}{l} a = \frac{2}{3h^2} \\ b = -1/h^2 \quad -\frac{3}{3h^2} \\ c = 1/3h^2 \end{array}$$

$$\boxed{f''(x) = \frac{2f(x) - 3f(x+h) + 1f(x+3h)}{3h^2} + \frac{4}{3}h\, f'''(\xi)}$$

$$\frac{27}{18} \quad 3$$

$$E = \frac{-1}{h^2} \cdot h \cdot \frac{h^3}{6} f'''(\xi) + \frac{1}{3h^2} \cdot \frac{27h^3}{6} f'''(\xi)$$

$$= -\frac{h}{6}f'''(\xi) + \frac{3h}{2}f'''(\xi) = \left(-\frac{1}{6} + \frac{9}{6}\right) h\, f'''(\xi)$$

$$= \frac{8}{6} h\, f'''(\xi) \boxed{= \frac{4h}{3} f'''(\xi)}$$