Score:

Name: Solution
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Team (circle one): a b c d e f

SM365 – Numerical Computing – Quiz 6 – Section 3.4
LU Factorization

1. Solve $\begin{cases} x_1 + 2x_2 = 1 \\ 3x_1 + 4x_2 = 2 \end{cases}$ using LU factorization. Do not pivot.

    a. Factorization Step:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{R2-3R1} \begin{bmatrix} 1 & 2 \\ (3) & -2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

    b. Solution Step:

$$A\vec{x} = \vec{b} \Rightarrow LU\underbrace{\vec{x}}_{\vec{z}} = \vec{b}$$

$$L\vec{z} = \vec{b} \Rightarrow \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \begin{array}{l} z_1 = 1 \\ z_2 = -1 \end{array}$$

$$U\vec{x} = \vec{z} \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \boxed{x_2 = \tfrac{1}{2}}$$

$$x_1 + 2x_2 = 1 \Rightarrow \boxed{x_1 = 0}$$