Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM365 – Numerical Computing – Quiz 4– Section 3.2
Matrix Norms

1. Let $\|\cdot\|_v$ be a vector norm. Show that the natural norm associated with $\|\cdot\|_v$ satisfies the property $\|AB\|_v \leq \|A\|_v\|B\|_v$ for all $A, B \in \mathbb{R}^{n\times n}$.

(1) $\|ABx\| = \|A(Bx)\|$ associative property of matrix multiplication

(2) $\|A(B\vec{x})\| \leq \|A\|\|B\vec{x}\| \leq \|A\|\|B\|\|\vec{x}\|$ consistency property of natural norm

(3) $\frac{\|AB\vec{x}\|}{\|\vec{x}\|} \leq \frac{\|A\|\|B\|\|\vec{x}\|}{\|\vec{x}\|} = \|A\|\|B\|$

(4) max value of $\frac{\|AB\vec{x}\|}{\|\vec{x}\|}$ for $\|\vec{x}\| \neq 0$ is $\|AB\|$ "definition of matrix norm"

(5) $\|AB\| \leq \|A\|\|B\|$ QED