Lecture Notes

Section 5.4b
Optimizing Points
Euclidian Norm
Legendre Polynomial

I) Review Chebyshev

A) SO FAR WE HAVE SHOWN THAT THE ROOTS OF THE CHEBYSHEV POLYNOMIAL CAN BE USED TO CREATE AN INTERPOLATING POLN $P_n(x)$

SUCH THAT $\| f(x) - P_n(x) \|_\infty$ IS minimized

the infinite norm

B) $$\| f(x) \|_\infty = \max_{x \in [a,b]} |f(x)|$$

$$\| f(x) \|_2 = \left(\int_a^b [f(x)]^2 dx \right)^{1/2}$$

the 2 or Euclidian Norm

II) Chebyshev Roots will not minimize

$$\| f(x) - P_n(x) \|_2$$

but roots of the Legendre Polynomial will

A) Legendre Poly

$$P_n(x) = \frac{2n-1}{n} P_{n-1}(x) - \frac{n-1}{n} P_{n-2}(x)$$

B) Let $P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{3}{2}x(x) - \frac{1}{2} = \frac{3}{2}x^2 - \frac{1}{2}$

$P_3(x) \quad \frac{5}{2}x^3 - \frac{3}{2}x$ etc.

<u>Note:</u> It is not as easy to get the roots as it was for Chebyshev polys, but w/ the proper methods we can find them numerically

C) EXAMPLES:

① $x\sin(\pi x)$ vs $P_5(x)$ on $[-1,1]$

HW('x.*sin(pi.*x)', [-1,1], 5