

I) Example: (A MIRACLE?)

(x_0, f_0) (x_1, f_1) (x_2, f_2)

a) Consider the Data Points: $(-2, -3)$, $(1, 0)$, $(2, 5)$

b) Make the following table:

x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
-2	-3	1	1
1	0	5	
2	5		

This is called a "divided difference table"

c) note: ① $f[x_i] = f_i$
"zeroth divided difference"

② $f[x_i, x_{i+1}] = \frac{f_{i+1} - f_i}{x_{i+1} - x_i}$
"1st div. diff"

③ $f[x_i, x_{i+1}, x_{i+2}] = \frac{f_{i+2} - f_i}{x_{i+2} - x_i}$
"2nd div. diff"

d) Now Consider the circled values

$$f[x_0], f[x_0, x_1], f[x_0, x_1, x_2]$$

$$P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$\Rightarrow P_2(x) = -3 + 1(x - (-2)) + 1(x - (-2))(x - 1)$$

$$\Rightarrow P_2(x) = -3 + x + 2 + x^2 + 2x - x - 2 \Rightarrow P_2(x) = x^2 + 2x - 3 \quad !!$$

II Add a point to the dataset Find $P_3(x)$

$$(-2, -3), (1, 0), (2, 5), (4, 15)$$

we can use the previous table!

	-2	-3			
"DIVIDED DIFFERENCE TABLE"	1	0	1	1	
	2	5	5	0	$[-1/6]$
			5		

$$\Rightarrow P_3 = -3 + 1(x+2) + 1(x+2)(x-1) - \frac{1}{6}(x+2)(x-1)(x-2)$$

same as before

$$\Rightarrow P_3 = -3 + x + 2 + x^2 + 2x - x - 2 - \frac{1}{6}x^3 + \frac{1}{6}x^2 + \frac{2}{3}x - \frac{2}{3}$$

$$\Rightarrow P_3 = -\frac{1}{6}x^3 + \frac{7}{6}x^2 + \frac{1}{3}x - \frac{11}{3}$$

Advantage of Newton over Lagrange ...

one can add new points without starting from scratch

Example

III Consider Follow Data Points:

x	f
0	3
1	5
2	7
3	9

(2) what's going on
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$$P_3 = 3 + 2(x-0) + 0(x-0)(x-1) + 0(x-0)(x-1)(x-2)$$

$$\Rightarrow \boxed{P_3(x) = 2x + 3}$$

conclusion... Data was linear!!

Recall: Theorems guarantee a polynomial of "at most" n

IV ERROR

$$f(x) - P(x) = f[x_0, x_1, x_2, \dots, x_n, x] \prod_{i=0}^n (x - x_i)$$

looks like Lagrange Error

... In fact

V Theorem (1) Let x_0, \dots, x_n be $n+1$ distinct points on $[a, b]$

(2) IF f is continuous on $[a, b]$

(3) w/ n continuous derivatives on (a, b)

(4) $\exists \xi$ on (a, b)

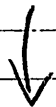
(5) $\Rightarrow f[x_0, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}$

A. Setting Up A Proof

① consider $f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

↑ looks like slope of secant line.

② By mean value theorem there \exists a $\xi \in (x_0, x_1)$ such that $\frac{f(x_1) - f(x_0)}{x_1 - x_0} = f'(\xi)$



\Rightarrow How might I extend this thought and prove

$$f[x_0, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$