Lecture Notes

Section 5.1

Interpolation (p 337-340)

I) Interpolation - Example Consider the following data table that reports the spread of an epidemic

t	$D(t)$
0	0
.75	446
1.50	843
2	1095

t = time in wks

$D(t)$ = # of people who have died

a) From table estimate how many died after 1 week

b) would probably try something like

$$D(1) = 446 + \frac{1-.75}{1.5-.75}(843-446) = 570.33 \approx 571$$

c) or in general, for $t_0 < t < t_1$

$$D(t) = D(t_0) + \frac{t-t_0}{t_1-t_0}\left(D(t_1) - D(t_0)\right)$$

or $\boxed{D(t) = \frac{t_1-t}{t_1-t_0}D(t_0) + \frac{t-t_0}{t_1-t_0}D(t_1)}$

Linear Interpolation

or $\boxed{D(t) = \frac{t-t_1}{t_0-t_1}D(t_0) + \frac{t-t_0}{t_1-t_0}D(t_1)}$

But was data really linear? 901

Note any structure to this

d) Linear Interpolation gives us a good estimate of what lies between known data. However there are other types of interpolation, i.e.

1. polynomial
2. piecewise polynomial
3. rational
4. trigometric
5. exponential

e) Interpolation vs approximation

a) Interpolation values are known at specific points, error is local to those points

b) error is global to the entire interval of interest

II) Fandament Problem:

Given a set of points (x_i, f_i) $i=1,2,\ldots n$,

1. approximate f for some value of x not listed

a 2. find a function g that approximates f

III Weierstrass Approximation Theorem

Let f be continues on $[a,b]$. Given $\epsilon > 0$, there exists a polynomial p such that

$$\|f-p\|_\infty = \max_{x\in[a,b]} |f(x)-p(x)| < \epsilon$$