Lecture Notes

Section 3.2
Pivoting Strategies

II **Example:**

$$\frac{1}{7}x + \frac{9}{49}y = \frac{16}{49}$$
$$\frac{1}{3}x + \frac{11}{9}y \quad \frac{14}{9}$$

$\boxed{\text{ans: } x=1,\ y=1}$

a) Use Gaussian Elimination, no pivots, 3 digits

$$\begin{bmatrix} .143 & .184 & .327 \\ .333 & 1.22 & 1.56 \end{bmatrix}$$

$$R2 - \frac{.333}{.143}R_1 \Rightarrow R2 - 2.33R$$

(1) $.333 - 2.33(.143) = .333 - .333 = 0$
(2) $1.22 - 2.33(.184) = 1.22 - .429 = .791$
(3) $1.56 - 2.33(.327) = 1.56 - .762 = .798$

$$\Rightarrow \begin{bmatrix} .143 & .184 & .327 \\ 0 & .791 & .798 \end{bmatrix}$$

$\Rightarrow .791y = .798 \Rightarrow \boxed{y = 1.01}$

$\Rightarrow .143x + .184(1.01) = .327$
$\Rightarrow .143x + .186 = .327$
$\Rightarrow x = .141/.143 \Rightarrow \boxed{x = .986}$

b) Gaussing Elimination - Partial Pivoting

1) - OF THE POSSIBLE PIVOTS IN THE COLUMN PICK THE ELEMENT WHOSE ABSOLUTE VALUE IS THE GREATEST, USE THAT AS THE PIVOT

(b) $\underline{\text{use}}$ $\boxed{\text{PARTIAL PIVOTING}}$, 3 digits

(Pick largest absolute value in the column)

$$\begin{bmatrix} .143 & .184 & .327 \\ .333 & 1.22 & 1.56 \end{bmatrix}$$

Pivot

$$R1 - \frac{.143}{.333} R_2 = .429$$

carefully perform each op @ 3 digits

(1) $.143 - (.429)(.333) = .143 - .143 = 0$ ✓

(2) $.184 - (.429)(1.22) = .184 - .523 = -.339$

(3) $.327 - (.429)(1.56) = .327 - .669 = -.342$

$$\Rightarrow \begin{bmatrix} 0 & -.339 & -.342 \\ .333 & 1.22 & 1.56 \end{bmatrix}$$

New Matrix

solve for y

$-.339y = -.342 \Rightarrow \boxed{y = 1.01}$

Solve for x

$.333x + 1.22(1.01) = 1.56$

$\Rightarrow .333x + 1.23 = 1.56$

$\Rightarrow .333x = .33 \Rightarrow \boxed{x = .991}$

Though y did not improve significantly we did see an improvement in X

Note the improvement in Round-off using partial pivoting.

c) Gaussian Elimination: Scaled Pivoting

1. Consider the "candidates" for pivots in the column $|a_i|$
2. Take the absolute value of that pivot and divide it by the absolute value of the row "member". Candidates include the potential pivot or anything to the right in Matrix A
3. calculate $\frac{|a_i|}{|s_i|}$ for each potential pivot.
4. use the element that generates the largest value $|a_i/s_i|$ as the pivot

Example:

$$\left[\begin{array}{cccc|c} 3 & 1 & 4 & -1 & 7 \\ 2 & -2 & -1 & -1 & 2 \\ 5 & 7 & 14 & -8 & 20 \\ 1 & 3 & 2 & 4 & -4 \end{array}\right]$$

PICK A PIVOT FOR COLUMN 1

a) WITHOUT PIVOTING: Pick 3

b) Partial Pivoting: Pick 5

c) Scaled Pivoting: $\frac{3}{4}$, $\frac{2}{2}$, $\frac{5}{14}$, $\frac{1}{4}$ $\rightarrow$ max in row 2 so pick 2

In Previous Example if I use 4 digit Precision:

① No Pivoting

$$X = \begin{bmatrix} \underset{x_1}{1.131}, & \underset{x_2}{-.7928}, & \underset{x_3}{.8500}, & \underset{x_4}{-.9987} \end{bmatrix}^T$$

② Partial Pivoting

$$x = [-1.000,\ +.9990,\ -.9985,\ 1.000]^T$$

③ Scaled Pivoting

$$X = [-1.000,\ 1.000,\ -1.000,\ 1.000]^T$$